Journal of Engineering Physics and Thermophysics, Vol. 67, No. 3-4, 1994

KINETIC MODEL OF THE ACTION OF A RAREFIED COLD-ION PLASMA FLOW ON A SPHERICAL METAL PARTICLE

A. G. Gnedovets, A. V. Gusarov, and A. A. Uglov

UDC 533.9

The processes of charge, momentum, and energy transfer to a spherical metal particle are considered in a rarefied cold-ion plasma flow.

The heat transfer and hydrodynamic resistance of material particles in plasma flows, as has been shown is a number of works [1–9], are largely determined by gas ionization, involvement of charge carriers – electrons and ions – in transfer processes, and electrization of the particles. Modeling the interaction of a particle with a plasma reduces in the general case to the solution of the Boltzmann–Vlasov kinetic equations for the velocity distribution functions and the Poisson equation for the electrostatic potential and can be performed only by numerical methods [9–11]. To analytically describe the plasma action as a function of whether the particle material is a semiconductor or a dielectric and as a function of the plasma velocity, the relation between the particle size, the Debye screening radius, and the mean free paths of the plasma particles, and the ratio of the electron and ion temperatures, use is made of different mathematical models [1–8] based on approximations.

The present work presents an analytical model of the processes of charge, momentum, and energy transfer to an individual (conducting) particle of spherical shape in a rarefied (collision-free) plasma flow under conditions where the thermal motion of the ions can be neglected compared to the flow velocity.

Theoretical Analysis. Let a rarefied plasma move relative to a spherical conducting particle with a velocity V that is much larger than the thermal velocity of the ions $\bar{v}_i \sim (kT_{i\infty}/m_i)^{1/2}$ but at the same time is substantially smaller than the thermal velocity of the electrons $\bar{v}_e \sim (kT_\infty/m_e)^{1/2}$ so that we can neglect the thermal spread of ion velocities compared to the directional component of their velocity as well as particle motion relative to the electron gas. These conditions correspond to plasma flows whose temperature ratio $\tau = T_{i\infty}/T_\infty$ and velocity ratio $s = V/(2kT_{e\infty}/m_i)^{1/2}$ are connected by the inequality $\tau^{1/2} << s$, i.e., since the speed of sound in the plasma is $c = [(kT_{e\infty} + \gamma kT_{i\infty})/m_i]^{1/2}$, we consider either two-temperature plasma flows with "cold" ($\tau \to 0$) ions over the entire velocity ratio range or supersonic ($M = [2/(1+\gamma\tau)]^{1/2}s >> 1$) plasma flows with similar temperatures of the charge carriers.

It is assumed that the velocities and temperatures of the heavy particles in the plasma (molecules and ions) coincide in the undisturbed region of the flow.

A material particle in the plasma undergoes collisions with molecules, electrons, and ions, which results in transmission of momentum, energy, and charge. Plasma electrons recombine on the surface and are absorbed by the particle, and ions are neutralized by material electrons and are scattered by the surface as neutral molecules. Owing to substantially different average thermal velocities of the plasma electrons and ions ($\bar{v}_e/\bar{v}_i$) >> 1) the particle becomes electrized: it acquires an excess negative charge (potential), and a local electric field forms close to it that retards electrons and accelerates ions in such a way that oppositely charged flows are equalized. Since the time of charge accumulation on the particle turns out to be extremely small compared to characteristic times of thermal and hydrodynamic processes [1–9], we consider the regime to be quasistationary with respect to the potential.

A. A. Baikov Institute of Metallurgy, Russian Academy of Sciences, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 67, Nos. 3-4, pp. 295-302, September-October, 1994. Original article submitted June 30, 1993.

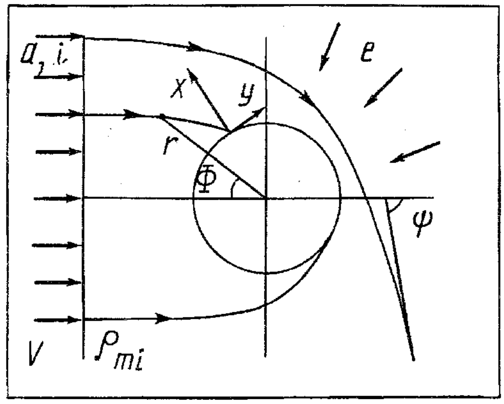


Fig. 1. System of coordinates.

Because of a high negative potential of the particle surface, which reflects most of the incident electrons, and a large ratio of the thermal electron velocity to the plasma flow velocity, the electron distribution close to the particle can be approximated by a Boltzmann one, for which the calculation of the charge and energy flows is performed, for example, in [2–9], and therefore the problem consists in describing the behavior of the ions. Formulas for molecular flows are a special case of the corresponding expressions for ions in the absence of the electric field.

In the considered case, when the thermal velocity of the heavy plasma particles turns out to be smaller than the ordered velocity, we need to describe the transfer of molecules and ions distributed regularly over the impact parameter ρ and moving initially toward the particle with the flow velocity V.

The chosen coordinate system is shown in Fig. 1.

We are able to express flows of the number of heavy plasma particles and their momentum and energy in terms of the characteristic cross sections of the interaction with a particle S_h^t, which include:

$$S_h^s = \pi \rho_{mh}^2 \tag{1}$$

is the direct-collision cross section, where ρ_{mh} is the maximum impact parameter for molecules or ions with trajectories incident on the particle surface;

$$S_h^r = 2\pi \int_0^{\rho_{mh}} \cos \Phi_1 \, \rho d\rho \tag{2}$$

is the reflection cross section, which characterizes the nonuniformity of the distribution of molecules and ions that collide with the particle over its surface; here Φ_1 is the angle between the internal normal at the incidence point and the direction of the flow;

$$S_i^C = 2\pi \int_{\rho_{mi}}^{\infty} (1 - \cos \psi) \, \rho d\rho \tag{3}$$

is the cross section of momentum transfer with Coulomb scattering of ions with trajectories not crossing the surface of the particle by its field; here ψ is the scattering angle.

The potential of the conducting-particle surface φ_f (floating potential) is determined from the condition of equality of the electron and ion flows that are collected by it:

$$I_e^- = I_i^- . \tag{4}$$

The total heat flux $Q = Q_a + Q_i + Q_e$ transferred from the plasma to the particle is comprised of molecular $Q_a = K_a^- - K_a^+$, ion $Q_i = K_i^- + I_i^-$, $W_i - K_i^+$, and electron $Q_e = \overline{K}_e + I_e^- W_e$ components and, apart from the kinetic energy of incident and reflected plasma particles, also includes the energy of the charge states of the ions $W_i = E_i - \Phi_e$ and electrons $W_e = \Phi_e$ released in collisions with the surface.

Flows of molecules and ions and their kinetic energy flows, arriving at the particle, are calculated as

$$I_h^- = \int_0^{\rho_{mh}} N_{h\infty} V 2\pi \rho d\rho = N_{h\infty} V S_h^s , \quad (5)$$

$$K_h^- = \int_0^{\rho_{mh}} N_{h\infty} V \left(\frac{1}{2} m_h V^2 - Z_h e\varphi_f \right) 2\pi\rho d\rho = N_{h\infty} V \left(\frac{1}{2} m_h V^2 - Z_h e\varphi_f \right) S_h^s , \quad (6)$$

where $Z_a = 0$, $Z_i = 1$.

The energy flow removed from the particle surface of temperature T_s is calculated in the following way:

$$K_h^+ = \int_{v_x > 0} v_x \frac{1}{2} m_h v^2 f_h^+ dv_x dv_y dv_z dS_p , \quad (7)$$

where the density $N_{h\infty}^+$ in the distribution function of diffusely reflected heavy particles of the plasma $f_h^+ = N_h^+ (m_h / 2\pi k T_s)^{3/2} \exp(-m_h v^2 / 2kT_s)$ is determined at each point of the surface from the condition of no accumulation of molecules and neutralized ions, $v^2 = v_x^2 + v_y^2 + v_z^2$, and the x axis of the Cartesian system of coordinates x, y, and z is normal to the surface. In the formula for K_h^+ we can pass from integration over the particle surface S_p to integration over the impact parameters ρ that had reflected molecules and ions prior to collision with the particle, which enables us to write

$$K_h^+ = \frac{2}{\pi} \left(\frac{m_h}{2kT_s} \right)^2 N_{h\infty} V \int_0^{\rho_{mh}} 2\pi\rho d\rho \int_{v_x > 0} v_x \frac{1}{2} m_h v^2 \times \exp\left(- \frac{m_h v^2}{2kT_s} \right) dv_x dv_y dv_z = 2kT_s I_h^- . \quad (8)$$

A contribution to the resistance force $F = F_a + F_i$ that acts on the particle is made only by the heavy plasma particles – molecules and ions – since electrons are distributed over the directions of motion almost isotropically and momenta transferred by them are mutually compensated. The neutral component $F_a = F_a^s + F_a^r$ is due to the collision and reflection of molecules from the surface while the ion one $F_i = F_i^s + F_i^r + F_i^C$ includes additionally a force connected with the Coulomb interaction of passing ions with the charged particle.

For direct collisions independently of whether there is interaction with the charged particle during the flight of the heavy plasma particles, as in the case of ions, or not, as in the case of molecules, the contribution to the resistance force is determined by the difference between their momentum in the undisturbed plasma region ($= m_h V$) and on the surface immediately after collision ($= 0$), and therefore

$$F_h^s = \int_0^{\rho_{mh}} N_{h\infty} V m_h V 2\pi\rho d\rho = m_h N_{h\infty} V^2 S_h^s . \quad (9)$$

The force exerted by the reflected molecules and neutralized ions is calculated in the following way:

$$F_h^r = \int_{v_x > 0} v_x m_h v_x \cos \Phi_1 f_h^+ dv_x dv_y dv_z dS_p =$$

$$= \frac{2}{\pi} \left(\frac{m_h}{2kT_s} \right)^2 N_{h\infty} V \int_0^{\rho_{mh}} 2\pi\rho d\rho \int_{v_x > 0} m_h v_x^2 \exp\left(- \frac{m_h v^2}{2kT_s} \right) dv_x dv_y dv_z =$$

$$= \frac{1}{2} \pi^{1/2} m_h (2kT_s / m_h)^{1/2} N_{h\infty} V S_h^r . \quad (10)$$

Momentum transfer by plasma ions with impact parameters $\rho > \rho_{mi}$ that do not collide with the surface is due to a change in their motion by the angle ψ after passing the charged particle. The corresponding component of the resistance force is equal to

$$F_i^C = \int_{\rho_{mi}}^{\infty} N_{i\infty} V m_i V (1 - \cos\psi)\, 2\pi\rho d\rho = m_i N_{i\infty} V^2 S_i^C . \tag{11}$$

Dimensionless Characteristics of Transfer Processes. An analysis of the influence of different factors (screening properties and velocity of the plasma flow, charge carrier temperatures, etc.) on the intensity of transfer processes can be made in the most general form if dimensionless characteristics of charge, energy, and momentum transfer are used:

$$i_j^{\pm} = I_j^{\pm}/4\pi R^2 N_{j\infty} (kT_{e\infty}/2\pi m_i)^{1/2}, \quad k_j^{\pm} = K_j^{\pm}/4\pi R^2 N_{j\infty} kT_{e\infty} (2kT_{e\infty}/\pi m_i)^{1/2},$$
$$q_j = Q_j/4\pi R^2 N_{j\infty} kT_{e\infty} (2kT_{e\infty}/\pi m_i)^{1/2}, \quad C_{Dj}^t = F_j^t/\left(\frac{1}{2} m_i N_{j\infty} V^2 \pi R^2\right). \tag{12}$$

For plasma electrons with a Boltzmann distribution all required moments of the distribution function can be expressed in terms of the floating potential of the particle φ_f, which enables us to write

$$i_e^- = \mu^{-1/2} \exp(-y_f), \quad k_e^- = \mu^{-1/2} \exp(-y_f), \tag{13}$$

where $\mu = m_e/m_i$, $y_f = -e\varphi_f/kT_{e\infty}$.

The dimensionless ion charge and kinetic energy flows are calculated as

$$i_i^- = \frac{1}{2}\pi^{1/2} s\sigma_i^s, \quad k_i^- = \frac{1}{4}\pi^{1/2} s (y_f + s^2)\sigma_i^s. \tag{14}$$

Here $s = V(2kT_{e\infty}/m_i)^{1/2}$, $\sigma_j^+ = S_j^t/\pi R^2$.

The dimensionless components of the heat flux are:

$$q_a = \frac{1}{2}\pi^{1/2} s\left(\frac{1}{2}s^2 - \tau_s\right), \quad q_i = k_i^- + i_i^-\left(\frac{1}{2}w_i - \tau_s\right), \tag{15}$$

where $\tau_s = T_s/T_{e\infty}$, $w_j = W_j/kT_{e\infty}$.

The components C_{Dj}^t of the drag coefficient are calculated as

$$q_e = k_e^- + \frac{1}{2} i_e^- w_e,$$

$$C_{Da}^s = 2, \quad C_{Da}^r = \frac{2}{3}\frac{(\pi\tau_s)^{1/2}}{s}, \quad C_{Di}^s = 2\sigma_i^s,$$

$$C_{Di}^r = \frac{(\pi\tau_s)^{1/2}}{s}\sigma_i^r, \quad C_{Di}^C = 2\sigma_i^C. \tag{16}$$

Limiting Cases. We are able to obtain analytical expressions for the cross sections of the interaction of ions with the particle, which determine completely the solution to the problem only in some cases when charge carrier motion can be considered in a field of central forces.

Since the metal particle surface is equipotential, then, as follows from the Poisson equation, the distribution of the potential around the particle is spherically symmetric if the condition of weak Debye screening $r_D \gg R$ is satisfied. We can also neglect the deviation from central symmetry when the field is weak throughout the bulk of the plasma, except, perhaps, for a thin layer of space charge that surrounds the particle, and it does not bend the trajectories of ion motion.

With the interaction energy $U_h(r) = Z_e e\varphi(r)$ the maximum impact parameters ρ_{mh} entering the expressions for the cross sections S_h^t are calculated in the approximation of weak screening:

$$\rho_{mh} = R\,(1 - 2Z_h e\varphi_f/m_h\,V^2)^{1/2} \tag{17}$$

and in the approximation of a weak field:

$$\rho_{mh} = R\,, \tag{18}$$

and the angles Φ_1 and ψ are determined as [12]

$$\Phi_1 = \Phi\,(\rho\,,\,R)\,,\quad \psi = \pi - 2\Phi\,(\rho\,,\,r_{mh})\,, \tag{19}$$

where the orientation angle $\Phi(\rho, r)$ is prescribed by the relation

$$\Phi\,(\rho\,,\,r) = \rho\int\limits_r^\infty [1 - 2U_h\,(r)/m_h\,V^2 - \rho^2/r^2]^{-1/2}\,r^{-2}\,dr\,, \tag{20}$$

and the distance of the closest approach r_{mh} is the largest real root of the equation $1 - 2U_h(r)/m_hV^2 - \rho^2/r^2 = 0$.

With weak Debye screening ($x_D = r_D/R >> 1$), by a method analogous to that used in [2, 8] and based on cutoff of the Coulomb potential $\varphi(r) = \varphi_f r/R$ at distances of about r_D from the particle, we can obtain

$$\sigma_i^s = 1 + g\,,\quad \sigma_i^r = \frac{2}{3}(1+g)^{3/2} + \frac{1}{2}g^2(1+g)^{1/2} - \frac{1}{2}g\,(1+g) - $$
$$- \frac{1}{2}g^2\left(1 + \frac{1}{2}g\right)\ln\left[1 + (1+g)^{1/2}\Big/\left(1 + \frac{1}{2}g\right)\right], \tag{21}$$
$$\sigma_i^C = \frac{1}{2}g^2\ln\left[\left(\frac{1}{4}g^2 + x_D^2\right)\Big/\left(\frac{1}{4}g^2 + \sigma_i^s\right)\right],$$

where $g = y_f/s^2$.

In the approximation of a weak field ($y << s^2$), which is valid either at supersonic flow velocities ($s >> 1$) or in the regime of strong Debye screening ($x_D = r_D/R << 1$), we can neglect the bending of the trajectories of ion motion, which enables us to write

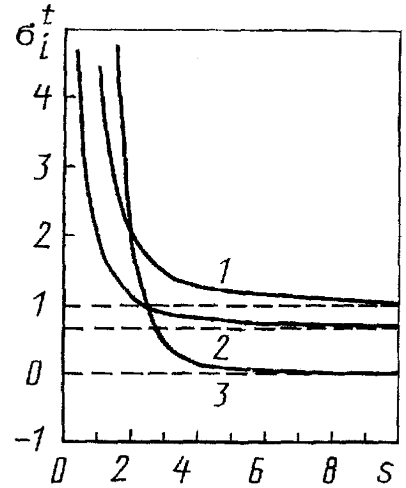
$$\sigma_i^s = 1\,,\quad \sigma_i^r = 2/3\,,\quad \sigma_i^C = 0\,. \tag{22}$$

Since the charged particle field has no effect on molecular motion the expressions for the cross sections σ_a^t coincide with (22).

Discussion of the Results. Figures 2–4 give the results of calculating the interaction of a spherical metal particle with an argon plasma flow. Since formulas for the approximation $x_D >> 1$ contain weak logarithmic dependences on the Debye parameter the data are plotted only for $x_D = 10$.

The intensity of the processes of charge, momentum, and energy transfer is ultimately determined by the behavior of cross sections of the interaction of the ions with the particle that are shown in Fig. 2. The electrostatic field of the charged particle, attracting the ions, acts on them more strongly, the smaller their kinetic energy. Therefore in a weakly screening plasma all the cross sections increase with a decrease in the velocity ratio and tend to the limiting values of the weak field approximation with an increase in it. Hence, with $s >> 1$ the influence of the screening properties of the plasma on the heat transfer and resistance of the particle in the flow should cease.

The drag coefficients of the particle C_{Di} are given in Fig. 3. The efficiency of momentum transfer by ions in a weakly screening plasma is much higher than by molecules owing to their interaction with the electrostatic



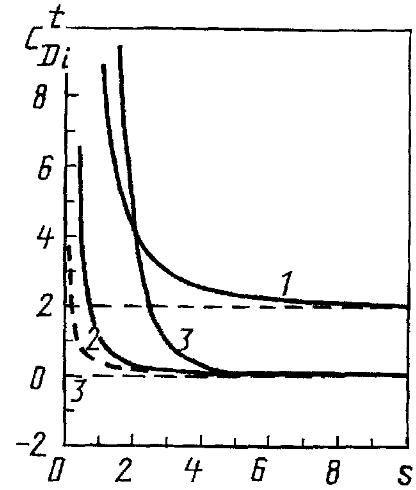


Fig. 2. Change in the dimensionless cross sections of the interaction of the ions with the particle σ_i^s (1), σ_i^r (2), and σ_i^C (3) vs velocity ratio s. The solid lines are the approximation of weak screening; the dashed ones are the approximation of a weak field.

Fig. 3. Drag coefficients C_{Di}^s (1), C_{Di}^r (2), and C_{Di}^C (3) vs velocity ratio s. The solid lines are the approximation of weak screening; the dashed ones are the approximation of a weak field. C_{Di}^r is calculated at $\tau_s = 0.1$.

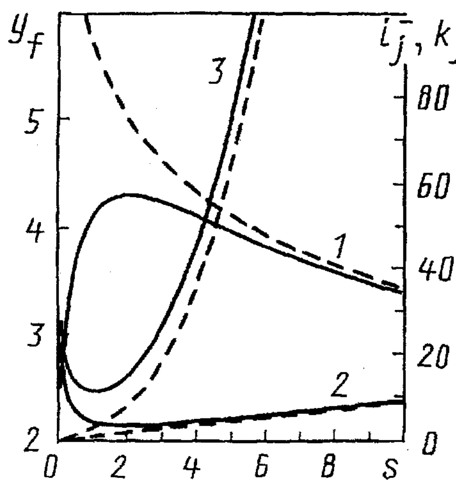


Fig. 4. Dimensionless floating potential of the particle y_f (1) and flows $i_i^- = i_e^- = k_e^-$ (2) and k_e^- (3) vs velocity ratio s. The solid lines are the approximation of weak screening; the dashed ones are the weak field approximation.

field of the charged particle, whose role is enhanced with a decrease in the velocity ratio s. Calculation shows that at large velocities the main contribution to the resistance force is made by ions that collide with the particle. In a weakly screening plasma with $s \lesssim 1$, when the influence of the field around the particle is substantial, it is the component connected with Coulomb scattering of the ions not colliding with the particle that prevails. The contribution of the ions reflected from the surface becomes noticeable only at low flow velocities.

The ions that collide with the surface neutralize partially the negative charge of the particle, and therefore the larger their flow the smaller the dimensionless floating potential y_f. As Fig. 3 shows, with an increase in the velocity ratio s the potential of the particle y_f decreases monotonically in the case of strong screening and has a local maximum at $s \simeq 1$ in the case of a weakly screening plasma. The presence of this maximum is due to competition between an increase in the velocity of the incident ions and a decrease in the cross section of direct collisions. It is important that with $s >> 1$ the ion flow of kinetic energy is much larger than the electron flow. The data in Fig. 3 determine completely the heat transfer of the particle with the plasma.

As follows from (15), the relation between the heat fluxes q_j transferred to the particle by molecules, electrons, and ions, depends strongly on the plasma velocity and is determined by the contribution of the corresponding flows of kinetic energy and charge state energies. For example, in an argon plasma with an electron temperature of about $kT_{e\infty} \approx 1$ eV, the dimensionless energies of charge states are equal respectively to $w_e \approx 5$ and $w_i \approx 10$. At rather low velocity ratios s both the ion heat flux $q_i \sim s\sigma_i^s(y_f + w_i)$ and the electron heat flux $q_e \sim s\sigma_j^s(1 + w_e/2)$ are much larger than the molecular heat flux $q_a \sim s^3$. In the region of high velocity ratios the ion and molecular heat fluxes are proportional to s^3 and prevail over the electron one, proportional to s. As a consequence of the release of charge state energies in electron recombination and ion neutralization on the surface and acceleration of ions in the electrostatic field of the particle the heat transfer by the charge carriers is much more efficient than molecular heat transfer.

Thus, gas ionization and particle electrization are of considerable importance in the processes of momentum and energy exchange between a particle and a rarefied plasma flow with cold ions. The relative contribution of the ions to the resistance force and of the electrons and ions to the heat flux is maximum in the region of not very high velocity ratios s in a weakly screening plasma. As the above calculations show, in the plasma flow with these parameters, $C_{Di} >> C_{Da}$ and $q_e + q_i >> q_a$, i.e., the charged components of the momentum and heat fluxes become comparable with the neutral ones even at small degree of ionization.

The model presented can be used to describe the motion and heating of micron-sized metal particles in plasma flows of reduced pressure. A comparison of the results of analytical models and numerical calculations [9] shows that at least in a resting plasma the approximation of a weak field is realized only with $x_D << 1$ while the approximation of weak Debye screening yields satisfactory agreement starting with $x_D \sim 1$.

NOTATION

c, speed of sound; C_{Dj}, drag coefficient; e, electron charge; E_i, ionization energy; F_j, resistance force; I_j, K_j, Q_j, flows of the number of plasma particles, kinetic energy, and heat; k, Boltzmann constant; m_j, mass; M, Mach number; N_j, number concentration; r, distance in spherical coordinates; r_D, Debye radius; R, particle radius; s, velocity ratio; S_j^t, interaction cross section; S_p, particle surface; T_j, temperature; V, plasma flow velocity with respect to the particle; x, y, z, Cartesian system of coordinates connected with an arbitrary point of the particle surface; γ, specific heat ratio; ρ, impact parameter; φ_j, floating potential of the particle; Φ, orientation angle; Φ_e, electronic work function. Subscripts and superscripts: a, molecules; e, electrons; i, ions; h, heavy plasma particles (molecules and ions); s, surface; ∞, value in the undisturbed region of the plasma away from the particle; + (–), direction from (toward) the particle.

REFERENCES

1. R. L. Boxman and S. Goldsmith, J. Appl. Phys., **52**, No. 1, 151-161 (1981).
2. A. A. Uglov, A. G. Gnedovets, and Yu. N. Lokhov, Teplofiz. Vys. Temp., **20**, No. 4, 621-626 (1982).
3. Y. C. Lee, Y. P. Chyou, and E. Pfender, Plasma Chemistry and Plasma Processing, **5**, No. 4, 391-414 (1985).
4. X. Chen and P. He, Plasma Chemistry and Plasma Processing, **6**, No. 4, 313-333 (1986).
5. X. Chen and X-g. Chen, Plasma Chemistry and Plasma Processing, **9**, No. 3, 387-408 (1989).
6. A. A. Uglov and A. G. Gnedovets, Teplofiz. Vys. Temp., **27**, No. 2, 247-253 (1989).
7. A. G. Gnedovets, Inzh.-Fiz. Zh., **58**, No. 1, 95-100 (1990).
8. A. A. Uglov and A. G. Gnedovets, Plasma Chemistry and Plasma Processing, **11**, No. 2, 251-267 (1991).
9. A. G. Gnedovets, A. V. Gusarov, and A. A. Uglov, Inzh.-Fiz. Zh., **62**, No. 2, 254-260 (1992).
10. A. V. Gurevich, L. P. Pitaevskii, and V. V. Smirnova, Usp. Fiz. Nauk, **99**, No. 1, 3-49 (1969).
11. P. Chan, L. Talbot, and K. Turyan, Electric Probes in Stationary and Moving Plasma [Russian translation], Moscow (1978).
12. I. McDaniel, Collision Processes in Ionized Gases [Russian translation], Moscow (1967).